Guide to Numeracy Across the Curriculum

Introduction

This information booklet has been produced as a guide for to make you more aware of how each topic is taught within the Maths Department.

It is hoped that the information in this booklet may lead to a more consistent approach to the use and teaching of Numeracy topics across the Learning Centre and consequently show an improvement in progress and attainment for all pupils.

I hope you find this guide useful.

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Addition

Mental Strategies

There are a number of useful mental strategies for addition. Some examples are given below.

Example: Calculate 34 + 49

Method 1 Add the tens, add the units, then add together

30 + 40 = 70   4 + 9 = 13   70 + 13 = 83

Method 2 Add the tens of the second number to the first and then add the units separately

34 + 40 = 74   74 + 9 = 83

Method 3 Round to the nearest ten, then subtract

34 + 50 = 84   (50 is 1 more than 49 so subtract 1)
84 – 1 = 83

Written Method

Before doing a calculation, pupils should be encouraged to make an estimate of the answer by rounding the numbers. They should also be encouraged to check if their answers are sensible in the context of the question.

Example: 3456 + 975

Line up the numbers according to place value

3 456 + 975

Estimate
3 500 + 1 000 = 4 500

	3	4	5	6
+	$_1$	9_1	7_1	5
	4	4	3	1

We add the numbers in each column from right to left

Carried numbers are written above the line

Subtraction

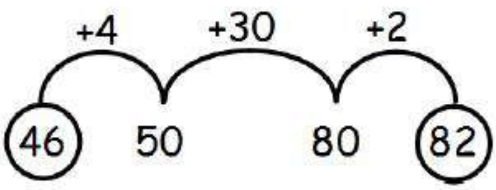
Mental Strategies

There are a number of useful mental strategies for subtraction. Some examples are given below.

Example: Calculate 82 - 46

Method 1 Start at the number you are subtracting and count on

e.g.



$\boxed{= 36}$

Method 2 Subtract the tens, then the units

82 - 40 = 42
42 - 6 = 36

Written Method

We use decomposition to perform written subtractions. We "exchange" tens for units etc rather than "borrow and pay back".

Before doing a calculation, pupils should be encouraged to make an estimate of the answer by rounding the numbers. They should also be encouraged to check if their answers are sensible in the context of the question.

Example:

Line up the numbers according to place value

6 286 – 4 857

Estimate
6 300 – 4 900 = 1 400

$$\begin{array}{rcccc} & {}^{5}\cancel{6} & {}^{1}2 & {}^{7}\cancel{8} & {}^{1}6 \\ - & 4 & 8 & 5 & 7 \\ \hline & 1 & 4 & 2 & 9 \end{array}$$

We subtract the numbers in each column from right to left

Multiplication

It is vital that all of the multiplication tables from 1 to 10 are known. These are shown in the multiplication square below:

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Mental Strategies

Example: Find 39×6

Method 1 Multiply the tens, multiply the units, then add the answers together

$30 \times 6 = 180$ $9 \times 6 = 54$ $180 + 24 = 234$

Method 2 Round the number you are multiplying, multiply and then subtract the extra

$40 \times 6 = 240$ (40 is one more than 39 so you have multiplied 6 by an extra 1)

$240 - 6 = 234$

Multiplication by 10, 100 and 1000

When multiplying numbers by 10, 100 and 1000 the **digits** move to **left**, we do not move the decimal point.

Multiplying by 10	-	Move every digit **one** place to the left
Multiplying by 100	-	Move every digit **two** places to the left
Multiplying by 1000	-	Move every digit **three** places to the left

Example 1

$46 \times 10 = 460$

Th	H	T	U
		4	6
	4	6	0

A zero is used to fill the gap in the units column

Example 2

$23 \times 100 = 2\,300$

Th	H	T	U
		2	3
2	3	0	0

Zeroes are used to fill gaps in the units and tens columns

Example 3 This rule also works for decimals

$3{\cdot}45 \times 10 = 34{\cdot}5$

Th	H	T	U	.	$\frac{1}{10}$	$\frac{1}{100}$
			3	.	4	5
		3	4	.	5	

The rule of simply adding zeros for multiplication by 10, 100 and 1000 can be confusing as it does not work for decimals and should therefore be avoided.

We can multiply by multiples of 10, 100 and 1000 using the same rules as above:

Example 4 Find 34×20 (multiply by 2 then by 10)

$34 \times 2 = 68$

$68 \times 10 = 680$

$68 \times 10 = 680$

Th	H	T	U
		6	8
	6	8	0

Multiplication by a Whole Number

When multiplying by a whole number, pupils should be encouraged to make an estimate first. This should help them to decide whether their answer is sensible or not.

Example

357 x 8

Estimate

$350 \times 2 \times 4 = 700 \times 4 = 2\,800$

$$\begin{array}{rrrr} & 3 & 5 & 7 \\ \times & {}_{4} & {}_{5} & 8 \\ \hline 2 & 8 & 5 & 6 \end{array}$$

Carried numbers go above the line

The number you are multiplying by goes under the last digit on the right

Example

329×42

Estimate

$300 \times 40 = 12\,000$

$$\begin{array}{rrrrr} & & 3 & 2 & 9 \\ & \times & 4_{1} & & 2 \\ \hline & {}^{1} & 6_{3} & 5 & 8 \\ 1 & 3 & 1_{1} & 6 & 0 \\ \hline 1 & 3 & 8 & 1 & 8 \\ \hline \end{array}$$

Carried numbers from multiplication by units go above the line

Multiply by the 2 first from right to left

Carried numbers from multiplication by tens go above digits

Put a zero before multiplying by TENS

Carried numbers from adding the answers together go above the line

Multiplication of a Decimal by a Decimal

We multiply decimals together by taking out the decimal points and performing a long multiplication:

Example 1 $0{\cdot}2 \times 0{\cdot}8$

Without the decimal points, the calculation is $2 \times 8 = 16$.
Each of the numbers ($0{\cdot}2 \times 0{\cdot}8$) have 1 decimal place, therefore the answer will have 2 decimal places, i.e. the total number of places after the point in the question.

So, $0{\cdot}2 \times 0{\cdot}8 = 0{\cdot}16$

Example 2 $2{\cdot}3 \times 4{\cdot}1$

$$\begin{array}{cccc}
 & & 2 & 3 \\
 & \times & 4 & 1 \\
\hline
 & & 2 & 3 \\
+ & 9 & {}^{1}2 & 0 \\
\hline
 & 9 & 4 & 3 \\
\hline
\end{array}$$

Each of the numbers ($2{\cdot}3$ and $4{\cdot}1$) have 1 decimal place, therefore the answer will have 2 decimal places.

So, $2{\cdot}3 \times 4{\cdot}1 = 9{\cdot}43$

Example 3 $0{\cdot}6 \times 5{\cdot}42$

$$\begin{array}{cccc}
 & 5 & 4 & 2 \\
\times & & & 6 \\
 & {}_{2} & {}_{1} & \\
\hline
3 & 2 & 5 & 2 \\
\hline
\end{array}$$

There are 3 decimal places altogether.

So, $0{\cdot}6 \times 5{\cdot}42 = 3{\cdot}252$

Division

Division by 10, 100 and 1000

When dividing numbers by 10, 100 and 1000 the **digits** move to **right**, <u>we do not move the decimal point</u>.

Dividing by 10	-	Move every digit **one** place to the right
Dividing by 100	-	Move every digit **two** places to the right
Dividing by 1000	-	Move every digit **three** places to the right

Example 1

$260 \div 10 = 26$

H	T	U	.	$\frac{1}{10}$	$\frac{1}{100}$
2	6	0	.		
	2	6	.		

Zeros are not generally needed in empty columns after the decimal point except in cases where a specified degree of accuracy is required

Example 2

$439 \div 100 = 4{\cdot}39$

H	T	U	.	$\frac{1}{10}$	$\frac{1}{100}$
4	3	9	.		
		4	.	3	9

Example 3 This rule also works for decimals

$32{\cdot}9 \div 10 = 3{\cdot}29$

H	T	U	.	$\frac{1}{10}$	$\frac{1}{100}$
	3	2	.	9	
		3	.	2	9

We can divide decimals by multiples of 10, 100 and 1000 using the same rules as discussed above.

Example 4 Find 48·6 ÷ 20

48·6 ÷ 2 = 24·3
24·3 ÷ 10 = 2·43

$24{\cdot}3 \div 10 = 2{\cdot}43$

H	T	U	.	$\frac{1}{10}$	$\frac{1}{100}$
	2	4	.	3	
		2	.	4	3

Division by a Whole Number

Example 1

$810 \div 6$

Estimate

$800 \div 5 = 160$

$$\begin{array}{r|l} & 1\ \ 3\ \ 5 \\ \hline 6 & 8\ {}^{2}1\ {}^{3}0 \end{array}$$

Example 2

When dividing a decimal by a whole number the decimal points must stay in line.

$$\begin{array}{r|l} & 4\cdot 1\ \ 3 \\ \hline 4 & 1\ {}^{1}7\cdot 2\ \ 4 \end{array}$$

Carry out a normal division

Example 3

If you have a remainder at the end of a calculation, add "trailing zeros" at the end of the decimal and keep going!

Calculate $2{\cdot}2 \div 8$

$$\begin{array}{r|l} & 0\cdot 2\ 7\ 5 \\ \hline 8 & 2\cdot {}^{2}2\,{}^{6}0\,{}^{4}0 \end{array}$$

"Trailing Zeros"

Division by a Decimal

When dividing by a decimal we use multiplication by 10, 100, 1000 etc to ensure that **the number we are dividing by becomes a whole number**.

Example 1 $24 \div 0.3$ (Multiply both numbers by 10)

$= 240 \div 3$
$= 80$

Example 2 $4{\cdot}268 \div 0{\cdot}2$ (Multiply both numbers by 10)

$= 42{\cdot}68 \div 2$
$= 21{\cdot}34$

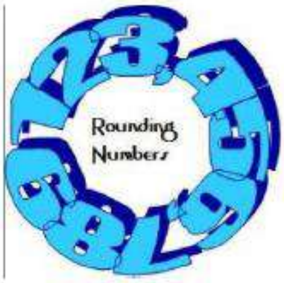
$$\begin{array}{r} 21\cdot34 \\ 2\overline{)42\cdot68} \end{array}$$

Example 3 $3{\cdot}6 \div 0{\cdot}04$ (Multiply both numbers by 100)

$= 360 \div 4$
$= 90$

Example 4 $52{\cdot}5 \div 0{\cdot}005$ (Multiply both numbers by 1000)

$= 52\,500 \div 5$
$= 10\,500$



Rounding

Numbers can be rounded to give an approximation.

The rules for rounding are as follows:

- less than 5 **ROUND DOWN**
- 5 or more **ROUND UP**

Example 1 Round the following to the nearest ten:

a) 34 b) 68 c) 283

Draw a dotted line after the number you are rounding to

The number beside the one you are rounding to helps you decide whether to round Up or DOWN

34 → 30 to the nearest ten

68 → 70 to the nearest ten

More than5 – ROUND UP

283 → 280 to the nearest ten

Example 2 Round the following to the nearest hundred

a) 267 b) 654 c) 2 393

267 → 300 to the nearest hundred

654 → 700 to the nearest hundred

2 393 → 2400 to the nearest hundred

Rounding Decimals

When rounding decimals to a specified decimal place we use the same rounding rules as before.

Example 1 Round the following to 1 decimal place:

a) 4·71 b) 23·29 c) 6·526

Draw a dotted line after the decimal place being rounded to

4 · 7 | 1 ⟶ 4 · 7 to one decimal place

2 3 · 2 | 9 ⟶ 23 · 3 to one decimal place

6 · 5 | 2 6 ⟶ 6 · 5 to one decimal place

The number in the next decimal place helps you to decide whether to round up or down

Example 2 Round the following to 2 decimal places:

a) 5·673 b) 41·187 c) 5·999

5 · 6 7 | 3 ⟶ 5 · 6 7 to two decimal places

4 1 · 1 8 | 7 ⟶ 41 · 1 9 to two decimal places

5 · 9 9 | 9 ⟶ 6 · 0 0 to two decimal places

Order of Operations

Care has to be taken when performing calculations involving more than one operation

e.g. $3 + 4 \times 2$ The answer is either $7 \times 2 = 14$ or $3 + 8 = 11$

The correct answer is 11.

Calculations should be performed in a particular order following the rules shown below:

B - Brackets

O - Order

D - Division
M - Multiplication
} Interchangeable

A - Addition
S - Subtraction
} Interchangeable

Most scientific calculators follow these rules however some basic calculators may not. It is therefore important to be careful when using them.

Example 1	$6 + 5 \times 7$ $= 6 + 35$ $= 41$	BODMAS tells us to multiply first
Example 2	$(6 + 5) \times 7$ $= 11 \times 7$ $= 77$	BODMAS tells us to work out the brackets first

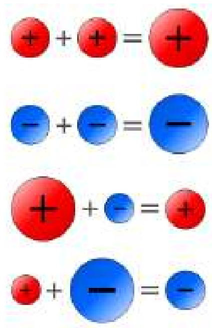
It is important to note that some of our pupils use BIDMAS instead, which is the same order but using 'Indices' (or powers) as the second word.

Example 3	$3 + 4^2 \div 8$	Order first (power)
	$= 3 + 16 \div 8$	Divide
	$= 3 + 2$	Add
	$= 5$	

Example 4	$2 \times 4 - 3 \times 4$	BODMAS tells us to multiply first
	$= 8 - 12$	
	$= -4$	

It is important to note that division and multiplication are interchangeable and so are addition and subtraction. This is particularly important for examples such as the following:

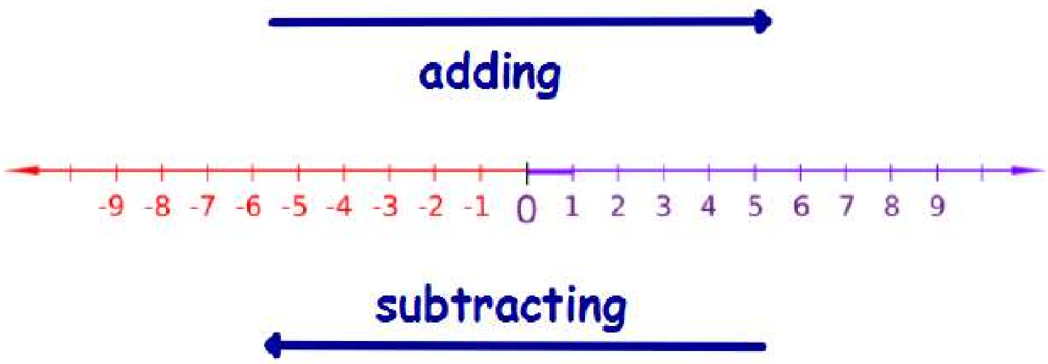
Example 5	$10 - 3 + 4$	In examples like this, go with the order of the question
	$= 11$	i.e. subtract 3 from 10 then add 4



Integers

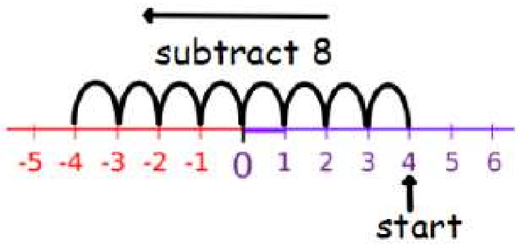
Integers are positive and negative whole numbers.
Negative numbers are numbers less than zero. They are referred to as "negative" numbers as opposed to "minus" numbers.

A number line often helps with integer calculations:

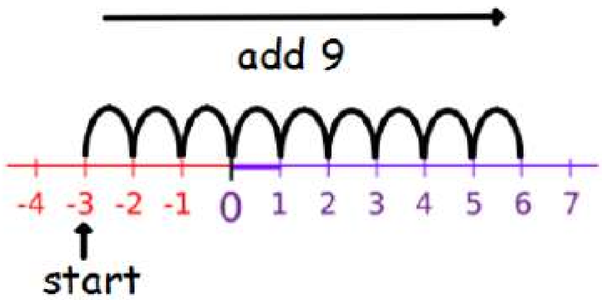


Examples

4 - 8 = - 4



- 3 + 9 = 6



Adding/Subtracting a Negative

When adding or subtracting a negative the following rules apply:

- Adding a negative is the same as subtracting

$5 + 5 = 10$
$5 + 4 = 9$
$5 + 3 = 8$
$5 + 2 = 7$
$5 + 1 = 6$
$5 + 0 = 5$
$5 + (-1) = 4$
$5 + (-2) = 3$
$5 + (-3) = 2$

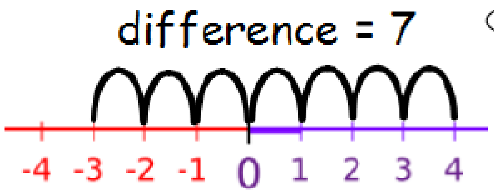
The pattern shows the answers decrease

Adding a negative has the same effect as subtracting

- Subtracting a negative is the same as adding

$$4 - (-3) = 7$$

When subtracting we are finding the difference



Examples

a) $2 + (-6)$
$= 2 - 6$
$= -4$

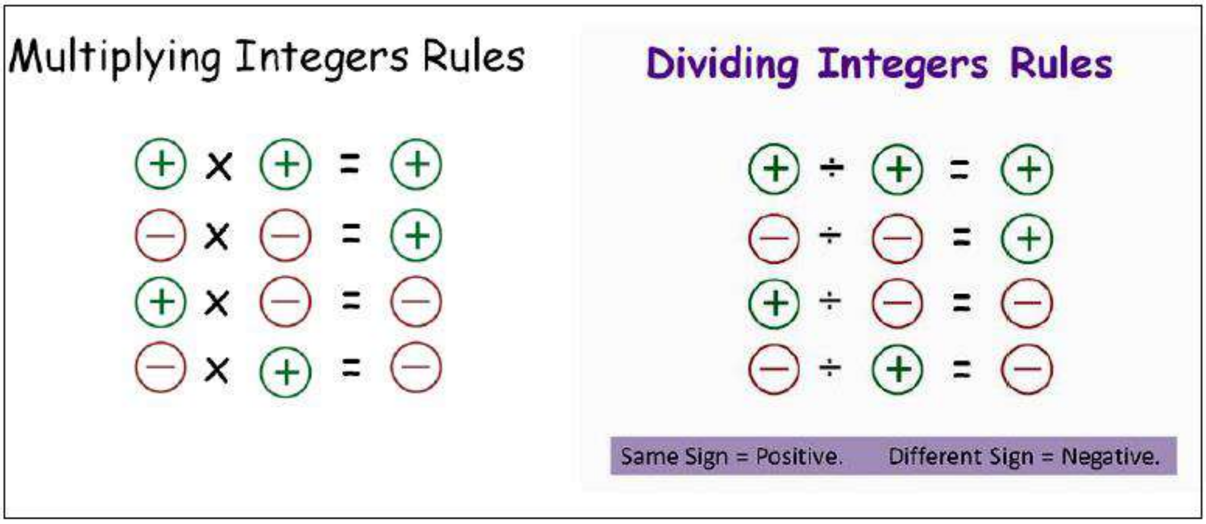
b) $-3 + (-5)$
$= -3 - 5$
$= -8$

c) $7 - (-4)$
$= 7 + 4$
$= 11$

d) $-2 - (-8)$
$= -2 + 8$
$= 6$

Multiplying/Dividing Integers

When multiplying and dividing integers the following rules apply:



Example 1

Same sign – positive answer

a) $5 \times 6 = 30$ b) $-5 \times (-6) = 30$

c) $5 \times (-6) = -30$ d) $-5 \times 6 = -30$

Different sign –negative answer

Example 2

a) $10 \div 5 = 2$ b) $-10 \div (-5) = 2$

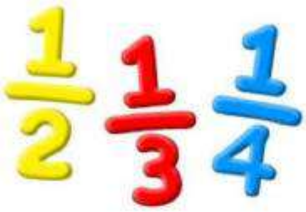
c) $10 \div (-5) = -2$ d) $-10 \div 5 = -2$

Example 3

a) $$(-6)^2 \\ = (-6) \times (-6) \\ = 36$$

b) $$-6^2 \\ = -(6 \times 6) \\ = -36$$

Fractions



Equivalent Fractions

Equivalent fractions are fractions which have the same value.
Examples of equivalent fractions are:

$$\underbrace{\overbrace{\frac{1}{2} \overset{\times 2}{\underset{\times 2}{=}} \frac{2}{4} = \frac{3}{6}}^{\times 3}}_{\times 3} = \frac{4}{8} = \frac{5}{10}$$

Equivalent fractions are found by multiplying the numerator and denominator by the same number

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15}$$

$$\frac{3}{5} = \frac{6}{10} = \frac{9}{15} = \frac{12}{20} = \frac{15}{25}$$

Simplifying Fractions

To simplify a fraction, divide the numerator and denominator by the **same number**.

Example 1

$$\frac{3}{12} \overset{\div 3}{\underset{\div 3}{=}} \frac{1}{4}$$

Example 2

$$\frac{15}{25} \overset{\div 5}{\underset{\div 5}{=}} \frac{3}{5}$$

In examples with higher numbers it is acceptable to use this process repeatedly in order to simplify fully.

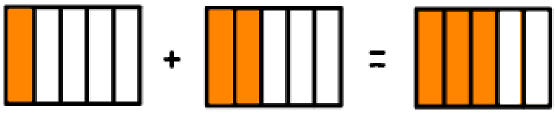
Example 3

$$\frac{48}{64} \overset{\div 2}{\underset{\div 2}{=}} \frac{24}{32} \overset{\div 4}{\underset{\div 4}{=}} \frac{6}{8} \overset{\div 2}{\underset{\div 2}{=}} \frac{3}{4}$$

Adding and Subtracting Fractions

When adding or subtracting fractions it is necessary to have a "common denominator".

e.g.



$$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$$

If this is the case then the numerators (top numbers) are simply added or subtracted.

When fractions have different denominators, equivalent fractions are used to obtain common denominators.

Example 1

Sometimes it is possible to turn one of the denominators into the other.
In this case 8 is the lowest common denominator

$$\frac{1}{4} + \frac{3}{8}$$

$$\times 2 \quad = \frac{2}{8} + \frac{3}{8}$$

$$= \frac{5}{8}$$

Example 2

We need to find the lowest common multiple of 2 and 3 in order to find the common denominator. In this case, the lowest common denominator is 6.

$$\frac{1}{2} + \frac{1}{3}$$

$$\times 3 \quad = \frac{3}{6} + \frac{2}{6} \quad \times 2$$

$$= \frac{5}{6}$$

When adding and subtracting mixed fractions, the fractions are changed to improper ("top heavy") fractions first.

Example 3

$$1\frac{1}{3} + 2\frac{2}{5}$$

$$= \frac{4}{3} + \frac{12}{5}$$

$$= \times 5 \quad \frac{20}{15} + \frac{36}{15} \quad \times 3$$

$$= \frac{56}{15}$$

$$= 3\frac{11}{15}$$

Mixed fractions should be changed into improper fractions before finding a common denominator. In this case the lowest common multiple of 3 and 5 is 15.

Multiplying and Dividing Fractions

To multiply fractions simply multiply the numerators together and the denominators together.

e.g.

$$\frac{2}{3} \times \frac{1}{5}$$

$$= \frac{2 \times 1}{3 \times 5}$$

$$= \frac{2}{15}$$

$$\frac{3}{4} \times \frac{2}{5}$$

$$\frac{3}{\cancel{4}_{2}} \times \frac{\cancel{2}^{1}}{5}$$

$$= \frac{3}{2} \times \frac{1}{5}$$

$$= \frac{3 \times 1}{2 \times 5}$$

$$= \frac{3}{10}$$

Fractions can be simplified prior to multiplying to keep the numbers as small as possible

Find one numerator and one denominator that are divisible by the same number – this is called "cancelling"

Example

$$1\frac{1}{3} + 2\frac{1}{4}$$

$$= \frac{4}{3} + \frac{9}{4}$$

$$= \times 4 \quad \frac{16}{12} + \frac{27}{12} \quad \times 3$$

$$= \frac{43}{12}$$

$$= 3\frac{7}{12}$$

As with adding and subtracting fractions, when multiplying mixed fractions they should be turned into improper fractions first.

An understanding of dividing fractions will be given in class and once understood the following quick method can be used:

- Flip the fraction you are dividing by upside down
- Multiply the fractions together
- Simplify where possible

Example

$$\frac{1}{3} \div \frac{2}{5}$$

$$= \frac{1}{3} \times \frac{5}{2}$$

$$= \frac{5}{6}$$

Fractions of a Quantity

To find a fraction of a quantity, divide by the denominator and multiply the answer by the numerator.

Example 1 Find $\frac{1}{3}$ of £120

$\frac{1}{3}$ of £120

= £120 ÷ 3

= £40

Divide by the numerator and multiply by the denominator

Example 2 Find $\frac{2}{5}$ of 200

$\frac{1}{5}$ of 200

= 200 ÷ 5

= 40

so $\frac{2}{5}$ of 200

= 40 x 2

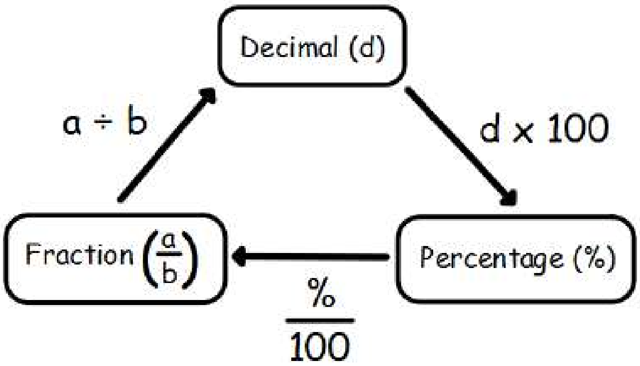
= 80

Percentages

Percent means "per hundred", i.e. out of 100.
A percentage can be converted to an equivalent fraction or decimal by dividing by 100.

24% means $\frac{24}{100}$

24% = $\frac{24}{100}$ = 0·24



Common Percentages

It is recommended that the information in the table below is learned.
The decimal and fraction equivalents of common percentages are used in percentage calculations.

Percentage	Fraction	Decimal
1%	$\frac{1}{100}$	0·01
5%	$\frac{1}{20}$	0·05
10%	$\frac{1}{10}$	0·1
20%	$\frac{1}{5}$	0·2
25%	$\frac{1}{4}$	0·25
$33\frac{1}{3}\%$	$\frac{1}{3}$	0·3333…
50%	$\frac{1}{2}$	0·5
$66\frac{2}{3}\%$	$\frac{2}{3}$	0·66666…
75%	$\frac{3}{4}$	0·75
100%	1	1

Calculating Percentages

Non-Calculator Methods

When calculating common percentages of a quantity, the fractional equivalents are used as follows:

Example 1 Find 25% of £240

25% of £240
$= \frac{1}{4}$ of £240
= £240 ÷ 4
= £60

Example 2 Find 20% of 180

20% of 180
$= \frac{1}{5}$ of 180
= 180 ÷ 5
= 36

More complicated percentages should be "broken down" into easier percentages as follows:

e.g. 35% = 25% + 10% OR 35% = (10% × 3) + 5%

35% = 1% × 35 OR 35% = 20% + 10% + 5%

The most appropriate method should be chosen depending on the numbers given.

Example 3 Find 65% of 2800

50% of 2800 = 2800 ÷ 2 = 1400
10% of 2800 = 2800 ÷ 10 = 280
5% of 2800 = 280 ÷ 2 = 140 (5% is half of 10%)

65% of 2800 = 1400 + 280 + 140 = 1820

It is also possible to find any percentage by first finding 1% .

Example 4 Find 24% of 3200

1% of 3200 = 3200 ÷ 100 = 32
24% of 3200 = 32 × 24
= 768

$$\begin{array}{r} 3\ 2 \\ \times\ \ 2\ 4 \\ \hline 1\ 2\ 8 \\ +\ 6\ 4\ 0 \\ \hline 7\ 6\ 8 \\ \hline \end{array}$$

Finding 17·5% (without a calculator)

Value Added Tax (VAT) used to be 17·5% (it is now 20%).
To calculate 17·5% without a calculator the following method is used:

- Find 10% first
- Find 5% by halving 10% value
- Find 2·5% by halving 5% value

Example Calculate the VAT on a computer costing £450.

10% of £450 = £450 ÷ 10 = £45 (divide by 10)
5% of £450 = £45 ÷ 2 = £22.50 (half previous answer)
2·5% of £450 = £22.50 ÷ 2 = £11.25 (half previous answer)

17·5% of £450 = £45 + £22.50 + £11.25
= £78.75

Therefore the VAT is £78.75

Calculator Method

To find a percentage of a quantity using a calculator, divide the percentage by 100 and multiply by the amount.

Example Find 23% of £15 000

$\frac{23}{100} \times$ £15 000
= 23 ÷ 100 × £15 000
= £3450

> We do NOT use the % button on calculators. We convert the percentage to a fraction, divide by 100 and then multiply

Expressing One Quantity as a Percentage of Another

You can express one quantity as a percentage of another as follows:

- Make a fraction
- Divide the numerator by the denominator
- Multiply by 100

Example 1 Ross scored 45 out of 60 in his Maths test. What is his percentage mark?

$\frac{45}{60}$ $45 \div 60 \times 100 = 75\%$

Example 2 There are 30 pupils in 1A2. 18 are girls.
What percentage of the pupils are girls?

$\frac{18}{30}$ $18 \div 30 \times 100 = 60\%$

Example 3 A survey of pupils' favourite sports was taken and the results were as follows:

Football – 11 Rugby – 3 Tennis – 4 Badminton – 2

What percentage of pupils chose tennis as their favourite sport?

Total number of pupils = 11 + 3 + 4 + 2 = 20
4 out of 20 pupils chose tennis

So, $\frac{4}{20}$ $4 \div 20 \times 100 = 20\%$

20% of pupils chose tennis as their favourite subject.

Ratio

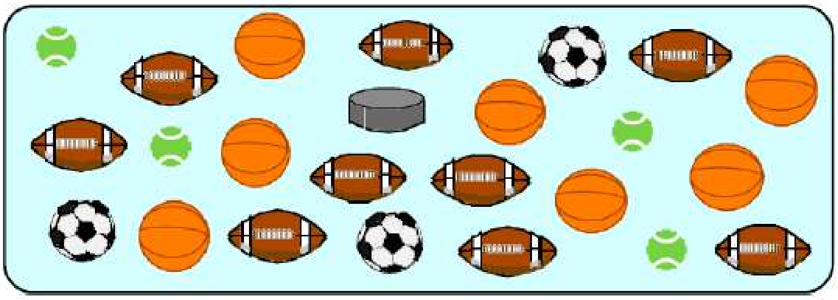
A ratio is a way of comparing amounts of something.
The ratio can be used to calculate the amount of each quantity or to share a total into parts.

Writing Ratios

The order is important when writing ratios.

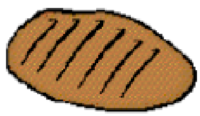
Example 1 For the diagram shown write down the ratio of

a) footballs : tennis balls
b) hockey pucks : basketballs



footballs : tennis balls	hockey pucks : basketballs
= 3 : 4	= 1 : 7

Example 2 In a baker shop there are 122 loaves, 169 rolls and 59 baguettes.



The ratio of loaves : baguettes : rolls is
122 : 59 : 169

Simplifying Ratios

Ratios can be simplified in much the same way as fractions by dividing all of the parts of the ratio by the same number

e.g. 12 : 6 : 3 can be simplified by dividing by 3 to get 4 : 2 : 1

Using Ratios

A given ratio can be used to find quantities by scaling up or down.

Example The ratio of boys to girls at a party is 2 : 3.
If there are 16 boys at the party, how many girls are there?

Put the ratio at the top

	boys	girls	
	2	3	
× 8	16	24	× 8

Find how the given value has been scaled up/down and do the same to the other side

So there are 24 girls at the party.

Sharing in a Given Ratio

Example Chris, Leigh and Clare win £900 in a competition.
They share their winnings in the ratio 2 : 3 : 4.
How much does each person receive?

1. Find the total number of shares

   2 + 3 + 4 = 9 i.e. there are 9 shares

2. Divide the amount by this number to find the value of each share

   £900 ÷ 9 = £100 i.e. each share is worth £100

3. Multiply each figure in the ratio by the value of each share

   2 shares: 2 x £100 = £200
   3 shares: 3 x £100 = £300
   4 shares: 4 x £100 = £400

4. Check that the total is correct by adding the values together

£200 + £300 + £400 = £900

So Chris receives £200, Leigh receives £300 and Clare receives £400.

Direct Proportion

Two quantities are said to be in direct proportion if when one quantity increases the other increases in the same way e.g. if one quantity doubles the other doubles.

When solving problems involving direct proportion **the first calculation is to find one of the quantities**.

Example 1 5 fish suppers costs £32.50, find the cost of 7 fish suppers.

The first calculation is always a divide

No. of fish suppers	Cost
5	£32.50
1	£32.50 ÷ 5 = £6.50
7	£6.50 x 7 = **£45.50**

Find the cost of one

So 7 fish suppers would cost £45.50.

Example 2 5 adult tickets for the cinema cost £27.50. How much would 8 tickets cost?

Tickets	Cost
5	£27.50
1	£27.50 ÷ 5 = £5.50
8	£5.50 x 8 = **£44.00**

The cost of 8 adult tickets is £44

Inverse Proportion

Two quantities are said to be in inverse proportion if when one quantity increases the other decreases e.g. when one quantity doubles the other halves.

When solving problems involving inverse proportion **the first calculation is to find one of the quantities**.

Example 1 If 3 men take 8 hours to build a wall, how long would it take 4 men to build the same wall?
(Common sense should tell us that it will take less time as there are more men working)

One man would take 24 hours

men	hours
3	8
1	$3 \times 8 = 24$
4	$24 \div 4 = 6$

The first calculation is always a multiply

4 men would take 6 hours to build the wall.

Example 2 An aeroplane takes 5 hours for a journey at an average speed of 500km/h.
At what speed would the aeroplane have to travel to cover the same journey in 4 hours?

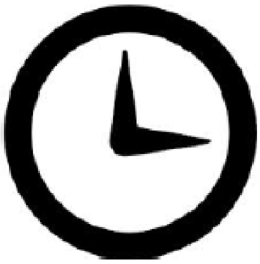
Find speed for 1 hour

Time (hours)	Speed (km/h)
5	500
1	$5 \times 500 = 2500$
4	$2500 \div 4 = 625$

Common sense - less time so more speed required!

The aeroplane would need to fly at an average speed of 625km/h

Time



12 hour Clock

Time can be displayed on a clock face or a digital clock.

When writing times in 12 hour clock we need to add a.m. or p.m. after the time.

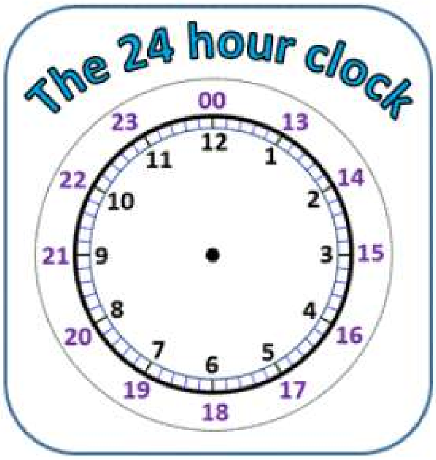
a.m. is used for times between midnight and 12 noon (morning) p.m. is used for times between 12 noon and midnight (afternoon/evening)

NOTE: 12 noon → 12.00 p.m.
12 midnight → 12.00 a.m.

24 hour Clock

When writing times in 24 hour clock a.m. and p.m. should not be used. Instead, four digits are used to write times in 24 hour clock.

After 12 noon, the hours are numbered 1300, 1400, . . . etc.



Examples

6.30 a.m. → 0630

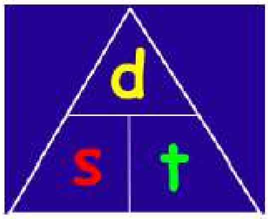
12.00 p.m. → 1200

2.45 p.m. → 1425

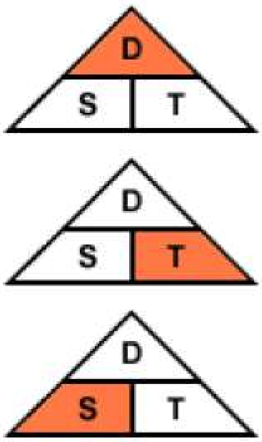
8.25 p.m. → 2025

12.00 a.m. → 0000

Distance, Speed and Time



The use of a triangle when calculating distance, speed and time will be familiar to pupils.



Distance = Speed x Time

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Example 1 A car travels at an average speed of 40mph for 5 hours. Calculate the distance covered.

S = 40 mph T = 5 hours

D = S × T
= 40 × 5
= 200 miles

Example 2 Calculate the average speed of a car which travels a distance of 168 miles in 3 hours and 30 mins.

D = 168 miles T = 3 hours 30 mins = 3·5 hours

S = D ÷ T
= 168 ÷ 3·5
= 48 mph

Time should be written as a decimal in SDT calculations

Example 3 Calculate the time taken for a car to travel a distance of 84 miles at an average speed of 35 mph.

D = 84 miles S = 35 mph

T = D ÷ S
= 84 ÷ 35
= 2·4 hours (= 2 hours 24 mins)

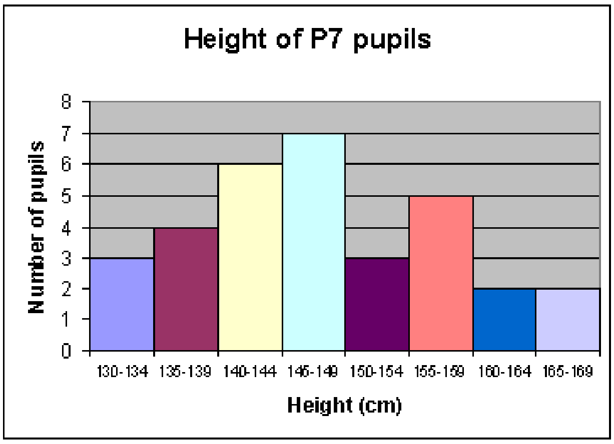
Change decimal time to hours and minutes by multiplying the decimal by 60 i.e. 0·4 × 60 = 24

Information Handling – Bar Graphs and Histograms

Bar graphs and histograms are often used to display information. The horizontal axis should show the categories or class intervals and the vertical axis should show the frequency.

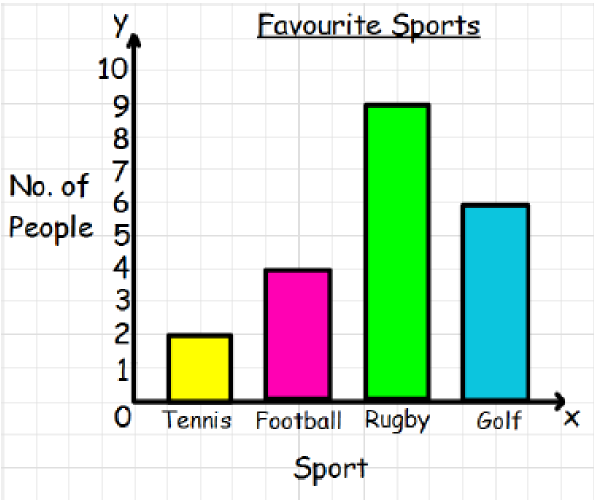
All graphs should have a title and each axis must be labelled.

Example 1 The **histogram** below shows the height of P7 pupils



Note that the histogram has **no gaps** between the bars as the data is continuous i.e. the scale has meaning at all values in between the ranges given. The intervals used must be evenly spaced (it must remain in this order).

Example 2 The **bar graph** below shows the results of a survey on favourite sports.



Note that the bar graph has gaps between the bars. The information displayed is non-numerical and **discrete** i.e there is no meaning between values (e.g. there is no in between for tennis and football). This means the order of the bars can be changed.

Examples of discrete data:

- Shoe Size
- Types of Pet
- Favourite subjects
- Method of travel to school

Examples of continuous data:

- Heights of pupils
- Weights
- Lengths of journeys to work
- Marks in a test

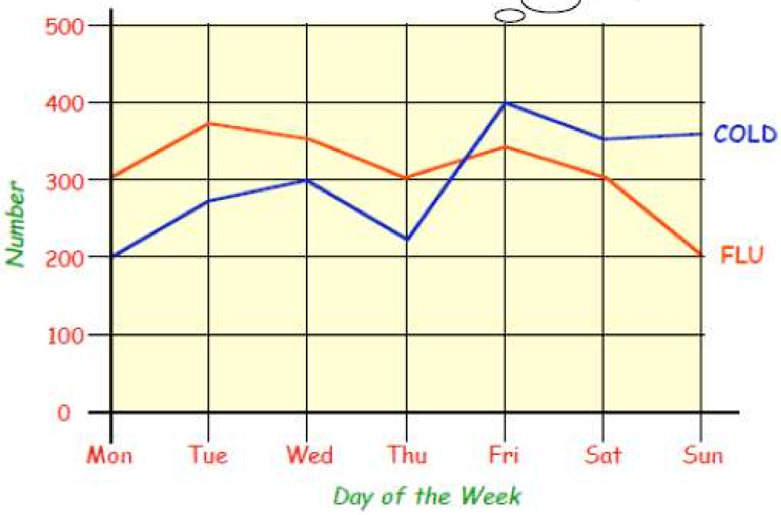
Information Handling – Line Graphs

Line graphs consist of a series of points which are plotted, then joined by a line. All graphs should have a title and each axis must be labelled. Numbers are written on a line and the scales are equally spaced and consistent. The trend of a graph is a general description of it.

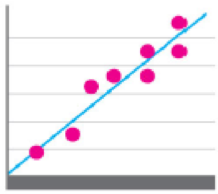
Example 1

Example 2

A comparative line graph can be used to compare data sets. Each line should be labelled or a key included.

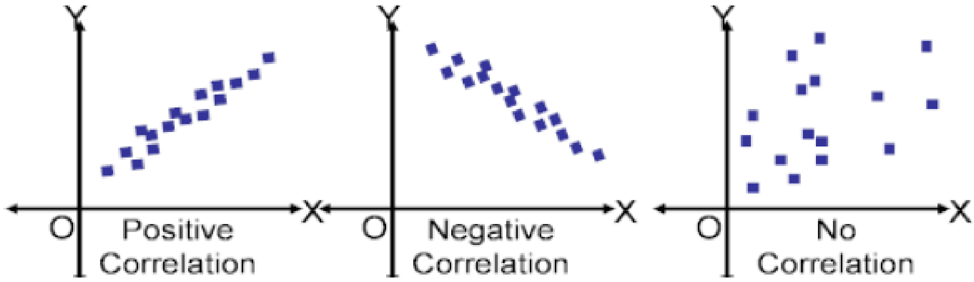


Information Handling – Scatter Graphs



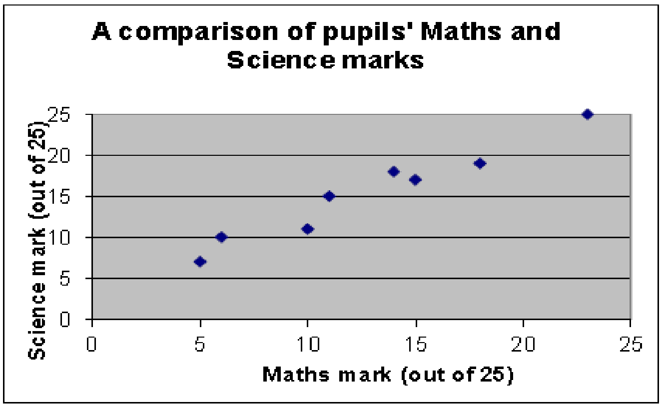
A scatter graph allows you to compare two quantities (or variables). Each variable is plotted along an axis. A scatter graph has a vertical and horizontal axis. It needs a title and appropriate x and y - axis labels. For each piece of data a point is plotted on the diagram. The points are not joined up.

A scatter graph allows you to see if there is a connection (correlation) between the two quantities. There may be a positive correlation when the two quantities increase together e.g. sale of umbrellas and rainfall. There may be a negative correlation where as one quantity increases the other decreases e.g. price of a car and the age of the car. There may be no correlation e.g. distance pupils travel to school and pupils' heights.



Example The table shows the marks gained by pupils in Maths and Science Tests. This information has been plotted on a scatter graph.

Maths Score	5	6	10	11	14	15	18	23
Science Score	7	10	11	15	18	17	19	25



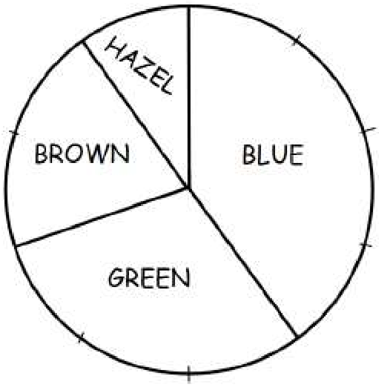
A **best-fit line** is meant to mimic the trend of the data. In many cases, the **line** may not pass through very many of the plotted points. Instead, the idea is to get a **line** that has equal numbers of points on either side.

Information Handling – Pie Charts

A pie chart can be used to display information.
Each sector of the pie chart represents a different category.
The size of each category can be worked out as a fraction of the total using the number of divisions or by measuring angles.

Using Fractions

Example 30 pupils were asked the colour of their eyes.
The results are shown in the pie chart below.



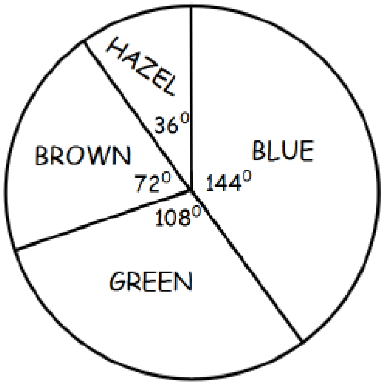
How many pupils had brown eyes?

The pie chart is divided up into ten equal parts, so pupils with brown eyes represent $\frac{2}{10}$ of the total.

$\frac{2}{10}$ of 30 = 6 (30 ÷ 10 x 2) so 6 pupils had brown eyes

Using Angles

If no divisions are marked on the pie chart and we are given the angles instead we can still work out the fraction by using the angle of each sector.



The angle of the brown sector is 72°. We can calculate the number of pupils as follows:

$$\boxed{\frac{angle}{360} \times total}$$

i.e. the number of pupils with brown eyes is

$$\frac{72}{360} \times 30 = 6$$

NB: Once you have found all of the values you can check your answers by making sure the total is 30.

Drawing Pie Charts

On a pie chart, the size of the angle for each sector is calculated as a fraction of 360°.

We calculate the angles as follows:

$$\boxed{\frac{amount}{total} \times 360}$$

Example In a survey about television programmes, a group of people were asked what their favourite soap was. Their answers are given in the table below. Draw a pie chart to illustrate this information.

Soap	Number of people
Eastenders	28
Coronation Street	24
Emmerdale	10
Hollyoaks	12
None	6

Step 1: Calculate the total number of people.

Total = 28 + 24 + 10 + 12 + 6 = 80

Step 2: Calculate the angles using the formula:

$$\boxed{\frac{amount}{total} \times 360}$$

Eastenders: $\frac{28}{80} \times 360^\circ = 126^\circ$

Coronation Street: $\frac{24}{80} \times 360^\circ = 108^\circ$

Emmerdale: $\frac{10}{80} \times 360^\circ = 45^\circ$

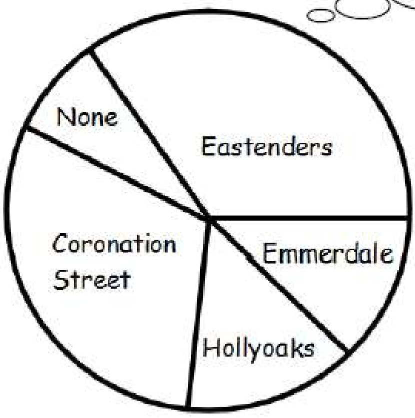
Hollyoaks: $\frac{12}{80} \times 360^\circ = 54^\circ$

None: $\frac{6}{80} \times 360^\circ = 27^\circ$

Always check that the angles add up to 360°.

Step 3: **Draw the pie chart**

Completed pie charts should be labelled or a key drawn to indicate what each sector represents.



Information Handling – Averages

To provide information about a set of data, the average value may be given. There are 3 ways of finding the average value – the mean, the median and the mode.

Mean

The mean is found by adding all of the values together and dividing by the number of values

e.g. 7 9 7 5 6 7 12 9 10

9 values in the set

Mean = (7 + 9 + 7 + 5 + 6 +7 + 12 + 9 + 8) ÷ 9
= 72 ÷ 9
= 8

Median

The median is the middle value when all of the data is written in numerical order (smallest to largest).

e.g. 7 9 7 5 6 7 12 9 10

Ordered list: 5 6 7 7 7 9 9 10 12

Median = 7

NOTE: If there are two values in the middle, the median is the mean of those two values.

e.g. 5 6 7 7 7 9 9 10 12 13

Median = (7 + 9) ÷ 2
= 16 ÷ 2
= 8

Mode

The mode is the value that occurs most often in the data set.

e.g. 5 6 $\boxed{7 \quad 7 \quad 7}$ 9 9 10 12

Mode = 7

Range

We can also calculate the range of a data set. This gives us a measure of spread.

e.g. 5 6 7 7 7 9 9 10 12

Range = highest value – lowest value
= 12 – 5
= 7

Evaluating Formulae

To find the value of a variable in a formula, we substitute all of the given values into the formula and use the BODMAS rules to work out the answer.

Example 1 Use the formula $P = 2L + 2B$ to evaluate P when $L = 12$ and $B = 7$.

Step 1: Write the formula	$P = 2L + 2B$
Step 2: Substitute numbers for letters	$P = 2 \times 12 + 2 \times 7$
Step 3: Start to evaluate (use BODMAS)	$P = 24 + 14$
Step 4: Write answer	$P = 38$

Example 2 Use the formula $I = \frac{V}{R}$ to evaluate I when $V = 240$ and $R = 40$.

Evaluate means find the value of . . .

$$I = \frac{V}{R}$$
$$I = \frac{240}{40}$$
$$I = 6$$

Example 3 Use the formula $F = 32 + 1\cdot 8C$ to evaluate F when $C = 20$.

$$F = 32 + 1\cdot 8C$$
$$F = 32 + 1\cdot 8 \times 20$$
$$F = 32 + 36$$
$$F = 68$$

$x + 4y + 6x + 2y$

Collecting Like Terms

An **expression** is a collective term for numbers, letters and operations

e.g. $3x + 2y - z$ $4m^2 + 5m - 1$

An expression does not contain an equals sign.

We can "tidy up" expressions by collecting "like terms". We circle letters which are the same (like) and simplify.

Example 1 Simplify $x + y + 3x$

Circle the "like terms" and collect them together.

$$x + y + 3x$$

$$= 4x + y$$

Don't forget to circle the sign as well!

Example 2 Simplify $2a + 3b + 6a - 2b$

Use a box for different like terms to make it stand out

$$2a \boxed{+\ 3b} + 6a \boxed{-\ 2b}$$

$$= 8a + b$$

Example 3 Simplify $2w^2 + 3w + \ + 3w^2 \ - w$

$$2w^2 \boxed{+\ 3w} + x + 3w^2 \boxed{-\ w}$$

$$= 5w^2 + 2w + x$$

Note that w^2 and w do not have the same exponents and are therefore not "like terms"

Solving Equations

An equation is an expression with an equals sign.

We solve equations by using a "method line". The method line is a list of steps taken in trying to solve an equation. When solving an equation we do the same to both sides of the equation in order to keep it **balanced**.

Basic Equations

Example 1 Solve $x+3=5$

$$\begin{array}{rl|l} x+3 & =5 & -3 \text{ from both sides} \\ x & =2 & \end{array}$$

We need to keep the equation balanced. To leave x on its own we have to subtract 3 and therefore we do this to both sides of the equation.

Example 2 Solve $k-2=6$

$$\begin{array}{rl|l} k-2 & =6 & +2 \text{ to both sides} \\ k & =8 & \end{array}$$

We are subtracting 2 from k so we have to add 2 to leave k on its own

Example 3 Solve $3p=9$

$$\begin{array}{rl|l} 3p & =9 & \div 3 \text{ on both sides} \\ p & =3 & \end{array}$$

p is being multiplied by 3 so we have to divide by 3 to solve for p.

Example 4 Solve $\frac{b}{4} = 2$

$$\frac{b}{4} = 2 \quad \Big| \times 4 \text{ on both sides}$$
$$b = 8$$

b is being divided by 4 so we have to multiply by 4 to solve for *b*

Two Step Equations

Example 1 Solve $2x + 1 = 9$

$$\begin{array}{rl|l} 2x + 1 &= 9 & -1 \\ 2x &= 8 & \div 2 \\ x &= 4 & \end{array}$$

"I'm thinking of a number. If I multiply it by 2 and add 1 I get 9.
What was the number I was thinking of?"

Think about working backwards!!

Example 2 Solve $5w - 2 = 8$

$$\begin{array}{rl|l} 5w - 2 &= 8 & +2 \\ 5w &= 10 & \div 5 \\ w &= 2 & \end{array}$$

Negative Letters

When solving equations with negative letters, the first priority is to get rid of them. We do this by adding the letters in as shown in the examples.

Example 1 Solve $10 - x = 7$

Remember, add x to both sides!

$$\begin{array}{ll|l} 10 - x & = 7 & +x \\ 10 & = 7 + x & -7 \\ 3 & = x & \end{array}$$

Example 2 Solve $16 - 2x = 8$

Adding $2x$ to both sides gets rid of the $-2x$ and leaves a positive $2x$ on the other side.

$$\begin{array}{ll|l} 16 - 2x & = 8 & +2x \\ 16 & = 8 + 2x & -8 \\ 8 & = 2x & \div 2 \\ 4 & = x & \end{array}$$

Letters on Both Sides

When solving equations with letters on both sides, the first step is to get rid of the smallest letter (adding it in if it is negative or subtracting it if it is positive).

Example 1 Solve $2x = x + 4$

x is smaller than $2x$, so we subtract x from both sides

$$\begin{array}{rr|l} 2x = & x + 4 & -x \\ x = & 4 & \end{array}$$

Example 2 Solve $3x+1=2x+5$

$$
\begin{array}{rcl|l}
3x+1 &=& 2x+5 & -2x \\
x+1 &=& 5 & -1 \\
x &=& 4 &
\end{array}
$$

Example 3 Solve $2x+9=4x-1$

$2x$ is smaller than $4x$

$$
\begin{array}{rcl|l}
2x+9 &=& 4x-1 & -2x \\
9 &=& 2x-1 & +1 \\
10 &=& 2x & \div 2 \\
5 &=& x &
\end{array}
$$

It is possible to solve equations with negative letters *and* letters on both sides in the same way.

Example 4 Solve $3x-4=8-x$

$-x$ is smaller than $3x$ so we get rid of negative letters and letters on both sides at the same time by adding x to both sides

$$
\begin{array}{rcl|l}
3x-4 &=& 8-x & +x \\
4x-4 &=& 8 & +4 \\
4x &=& 12 & \div 4 \\
x &=& 3 &
\end{array}
$$

Mathematical Dictionary (Key Words)

Add; Addition (+)	To combine two or more numbers to get one number (called the sum or the total) e.g. $23 + 34 = 57$
a.m.	(ante meridiem) Any time in the morning (between midnight and 12 noon).
Approximate	An estimated answer, often obtained by rounding to the nearest 10, 100, 1000 or decimal place.
Calculate	Find the answer to a problem (this does not mean that you must use a calculator!).
Data	A collection of information (may include facts, numbers or measurements).
Denominator	The bottom number in a fraction (the number of parts into which the whole is split).
Difference (-)	The amount between two numbers (subtraction). e.g. the difference between 18 and 7 is 11 $18 - 7 = 11$
Division (÷)	Sharing into equal parts e.g. $24 \div 6 = 4$
Double	Multiply by 2.
Equals (=)	The same amount as.
Equivalent fractions	Fractions which have the same vale e.g. $\frac{4}{8}$ and $\frac{1}{2}$ are equivalent fractions.
Estimate	To make an approximate or rough answer, often by rounding.
Evaluate	To work out the answer/find the value of.
Even	A number that is divisible by 2. Even numbers end in 0, 2, 4, 6, or 8.
Factor	A number which divides exactly into another number, leaving no remainder. e.g. The factors of 15 are 1, 3, 5 and 15.
Frequency	How often something happens. In a set of data, the number of times a number or category occurs.
Greater than (>)	Is bigger or more than e.g. 10 is greater than 6 i.e. $10 > 6$
Greater than or equal to (≥)	Is bigger than <u>OR</u> equal to.
Least	The lowest (minimum).

Less than (<)	Is smaller or lower than e.g. 15 is less than 21 i.e. 15 < 21
Less than or equal to (≤)	Is smaller than <u>OR</u> equal to.
Maximum	The largest or highest number in a group.
Mean	The arithmetic average of a set of numbers (see pg 43).
Median	Another type of average – the middle number of an ordered data set (see pg 43).
Minimum	The smallest or lowest number in a group.
Minus (-)	To subtract.
Mode	Another type of average – the most frequent number or category (see pg 44).
Most	The largest or highest number in a group (maximum).
Multiple	A number which can be divided by a particular number leaving no remainder e.g. the multiples of 3 are 3, 6, 9, 12, ...
Multiply (×)	To combine an amount a particular number of times e.g. 6 × 4 = 24
Negative Number	A number less than zero e.g. – 3 is a negative number.
Numerator	The top number in a fraction.
Odd Number	A number which is not divisible by 2. Odd numbers end in 1, 3, 5, 7 or 9.
Operations	The four basic operations are: addition, subtraction, multiplication and division.
Order of Operations	The order in which operations should be carried out (BODMAS)
Place Value	The value of a digit depending on its place in the number e.g. 1 342 - the number 4 is in the tens column and represents 40
p.m.	(post meridiem) Anytime in the afternoon or evening (between 12 noon and midnight).
Polygon	A 2D shape which has 3 or more straight sides.
Prime number	A number that has exactly 2 factors (can only be divided by itself and 1). Note that 1 is not prime as it only has one factor.
Product	The answer when two numbers are multiplied together e.g. the product of 4 and 5 is 20.

Quadrilateral	A polygon with 4 sides.
Quotient	The number resulting by dividing one number by another e.g. $20 \div 10 = 2$, the quotient is 2.
Remainder	The amount left over when dividing a number by one which is not a factor.
Share	To divide into equal groups.
Sum	The total of a group of numbers (found by adding).
Square Numbers	A number that results from multiplying a number by itself e.g. $6^2 = 6 \times 6 = 36$.
Total	The sum of a group of numbers (found by adding).

Useful websites

There are many valuable online sites that can offer help and more practice. Many are presented in a games format to make it more enjoyable.

The following sites may be found useful:

www.amathsdictionaryforkids.com

www.woodland-juniorschool.kent.sch.uk

www.bbc.co.uk/schools/bitesize

www.topmarks.co.uk

www.primaryresources.co.uk/maths

www.mathsisfun.com